Automated Market Makers 101
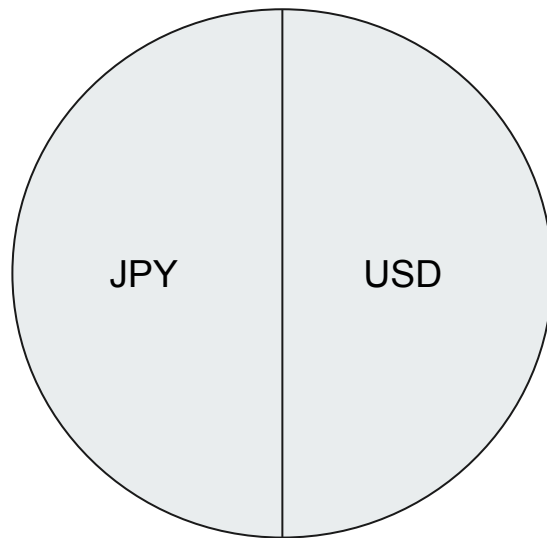
This talk

1. What is market making?
2. x*y=k, a simple yet elegant Constant Product Market Maker(CPMM)
3. Liquidity Pools: What does adding/removing liquidity mean?

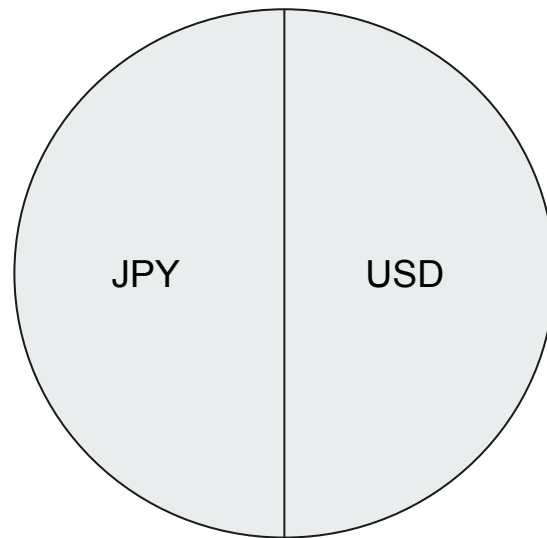# What does a currency exchange in an airport do?

What would you do if you run a currency exchange?

JPY   USD

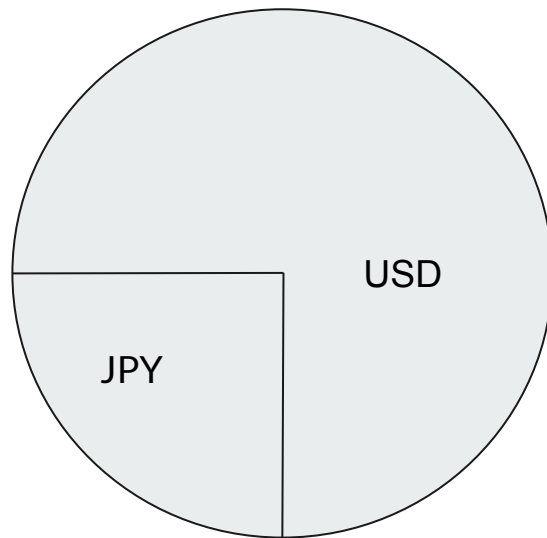2m JPY   70k USD

A flight from the US arrives

JPY | USD

2m JPY | 70k USD

35k USD (≈1.02m JPY)

You sold 1m JPY and got 35k USD

35k USD (≈1.02m JPY)

2m -1m = 1m JPY   70k+35k=105k

What if another flight arrives?

USD

JPY

1m JPY   105k USD

Raise the price for JPY

JPY 📈 📈 📈

USD

JPY
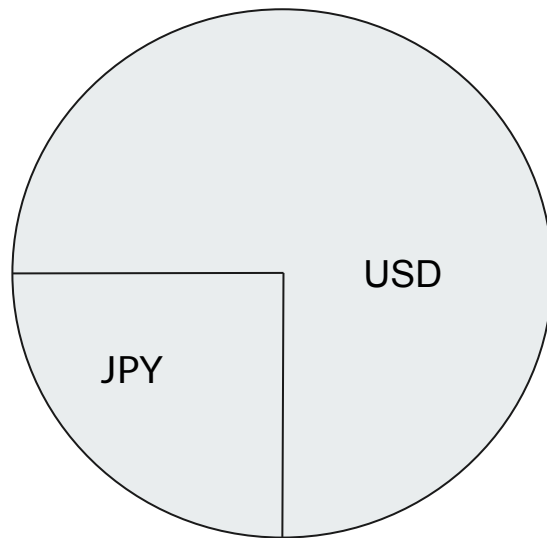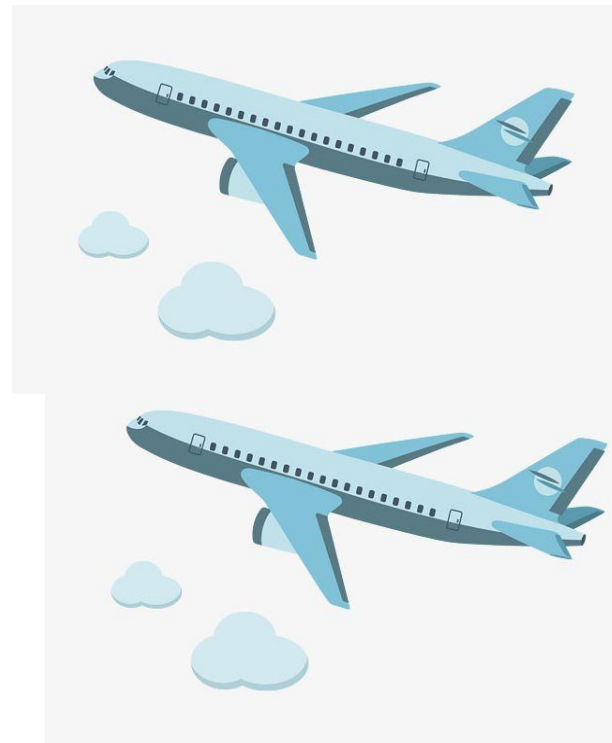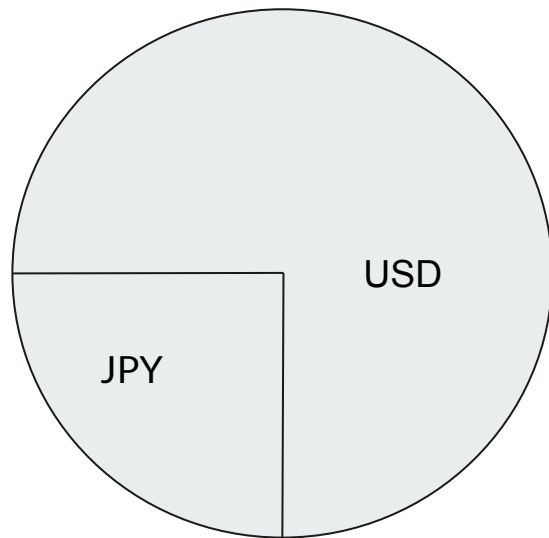
1m JPY   105k USD

Raise the price for JPY, but by how much?

JPY 📈📈📈

USD

JPY
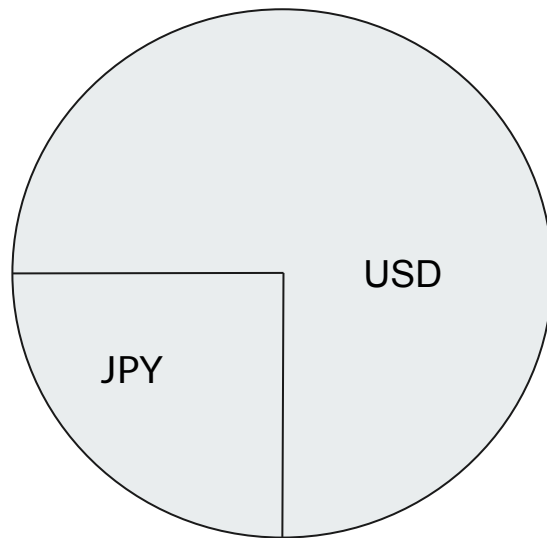
1m JPY   105k USD

What is Market Making?

1. **Traditional Commerce**

   E.g. Airport currency exchange

2. Traditional Finance

   E.g. Citadel Securities

3. Decentralized Finance

   E.g. Uniswap

What is Market Making?

1. Traditional Commerce

   E.g. Airport currency exchange

2. **Traditional Finance**

   E.g. Citadel Securities

3. Decentralized Finance

   E.g. Uniswap

Your Designated Market Maker

CITADEL SECURITIES DMM

We are a market leader in US equities, trading more than 22% in the U.S. listed equities market

What is Market Making?

1. Traditional Commerce

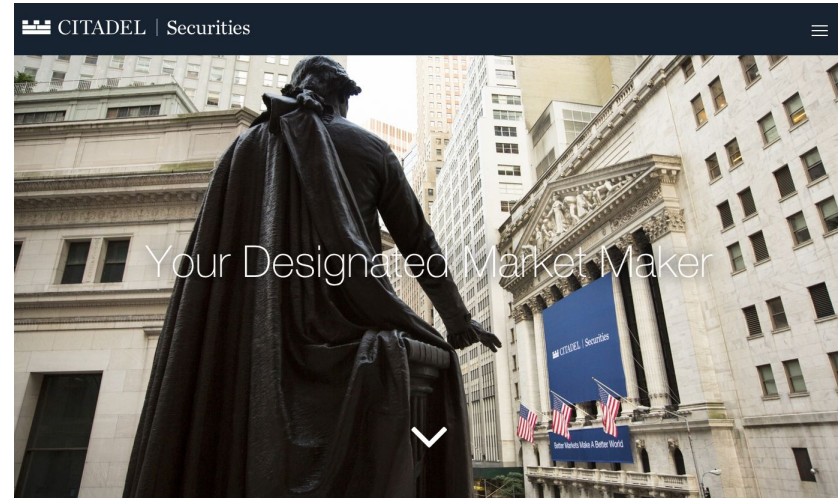   E.g. Airport currency exchange

2. **Traditional Finance**

   E.g. Citadel Securities

3. Decentralized Finance

   E.g. Uniswap

CITADEL | Securities

ABOUT CITADEL SECURITIES   PRODUCTS   PUBLIC POLICY

CAREERS

Making Tomorrow's Markets

Use strategies (e.g. delta neutrality) to manage risk

What is Market Making?

1. Traditional Commerce

    E.g. Airport currency exchange

2. Traditional Finance

    E.g. Citadel Securities

3. **Decentralized Finance**

    E.g. Uniswap and more

A fundamental question for all Market Makers

How do I price an asset any any given time?

A simple pricing function: maps the quantities of the assets to their marginal price

On-chain Automated Market Making

What is an Automated Market Maker(AMM)?

A robotic market maker that is **always willing to quote prices** between two assets A and B, **according to some pricing algorithm**

On-chain Automated Market Making

A robotic market maker that is **always willing to quote prices** between two assets A and B, **according to some pricing algorithm**
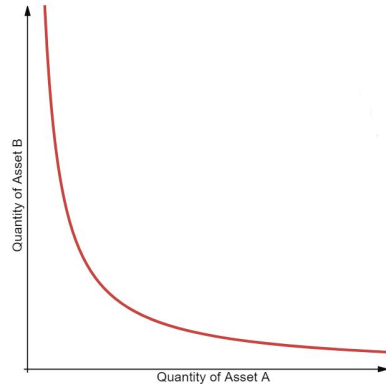
1. Always willing to provide price quotes **under any market conditions**
   - **permissioned variants exist that halts and withdraws liquidity during high volatility**
   - **We only focus on the permissionless ones in this talk**
2. The quote is provided according to some **pricing algorithm**

On-chain Automated Market Making

A robotic market maker that is **always willing to quote prices** between two assets A and B, **according to some pricing algorithm**

1. Always willing to provide price quotes **under any market conditions**
2. The quote is provided according to some **pricing algorithm**

💡 **x * y = k**

Constant Product Market Making by example

Suppose we have a trading pair WETH - WBTC

UNISWAP V2

Constant Product Market Making by example

Suppose we have a trading pair WETH - WBTC

Assume the exchange rate between WETH - WBTC is: 1 WBTC = 20 WETH

UNISWAP V2

Constant Product Market Making by example

Suppose we have a trading pair WETH - WBTC

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The pair(liquidity pool) is funded with 20 WBTC and 400 WETH

UNISWAP V2

Constant Product Market Making by example

Suppose we have a trading pair WETH - WBTC

Assume the exchange rate between WETH - WBTC is: 1 WBTC = 20 WETH

The pair(liquidity pool) is funded with 20 WBTC and 400 WETH   (x=20, y = 400)

The x*y=k (Constant Product Rule) says: k= 20*400=8000

UNISWAP V2

Constant Product Market Making by example

Suppose we have a trading pair WETH - WBTC

Assume the exchange rate between WETH - WBTC is: 1 WBTC = 20 WETH

The pair(liquidity pool) is funded with 20 WBTC and 400 WETH   (x=20, y = 400)

The x*y=k (Constant Product Rule) says: k= 20*400=8000

(Assuming no one adds/removes liquidity from this pair)

(For pedagogical purposes, we also assume 0 fees)

All trading(token swaps) MUST obey the x*y=k rule, enforced by the smart contracts

UNISWAP V2

Constant Product Market Making by example

20 WBTC and 400 WETH   (x=20, y = 400)   k= 20*400=8000

All trading MUST obey the x*y=k rule

If I want to sell 1 WBTC

UNISWAP V2

Constant Product Market Making by example

20 WBTC and 400 WETH   (x=20, y = 400)   k= 20*400=8000

All trading MUST obey the x*y=k rule

If I want to sell 1 WBTC

x: 20 ->21

UNISWAP V2

Constant Product Market Making by example

20 WBTC and 400 WETH   (x=20, y = 400)   k= 20*400=8000

All trading MUST obey the x*y=k rule

If I want to sell 1 WBTC

x: 20 ->21

y= k/x = 8000/21 ≈ 381

UNISWAP V2

Constant Product Market Making by example

20 WBTC and 400 WETH   (x=20, y = 400)   k= 20*400=8000

All trading MUST obey the x*y=k rule

If I want to sell 1 WBTC

x: 20 ->21

y= k/x = 8000/21 ≈ 381   This means WETH must 400 ->381

UNISWAP V2

Constant Product Market Making by example

20 WBTC and 400 WETH   (x=20, y = 400)   k= 20*400=8000

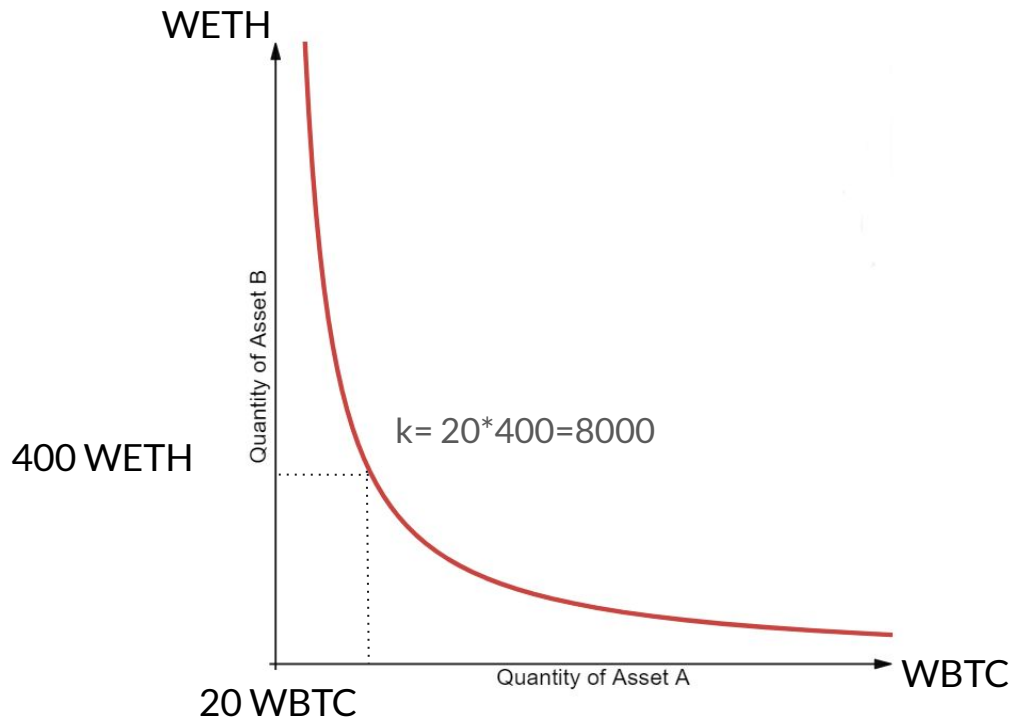**All trading MUST obey the x*y=k rule**

If I want to sell 1 WBTC

x: 20 ->21

y= k/x = 8000/21 ≈ 381   This means WETH must 400 ->381

I sold my 1 WBTC for 19 ETH (38,000 USD)

UNISWAP V2

Constant Product Market Making by example

WETH

400 WETH

20 WBTC

WBTC

Quantity of Asset B

Quantity of Asset A

k= 20*400=8000

UNISWAP V2

Constant Product Market Making by example

WETH

400 WETH

↓

381 WETH

k= 20*400=8000

k= 21*381=8000

Quantity of Asset B

Quantity of Asset A

WBTC

20 WBTC → 21 WBTC

UNISWAP V2
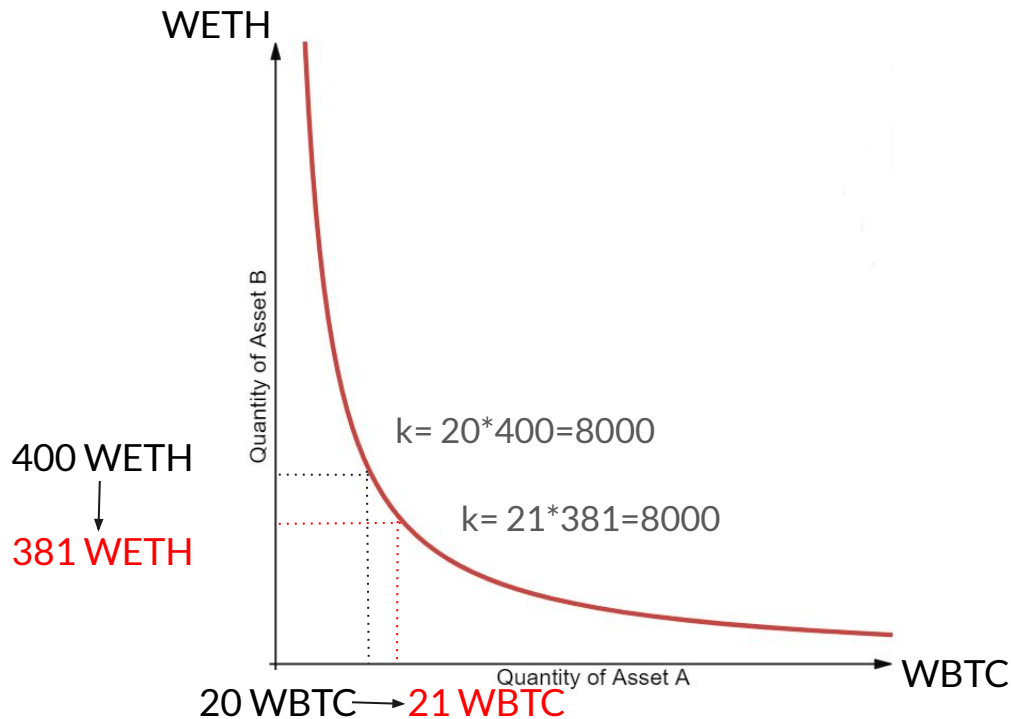
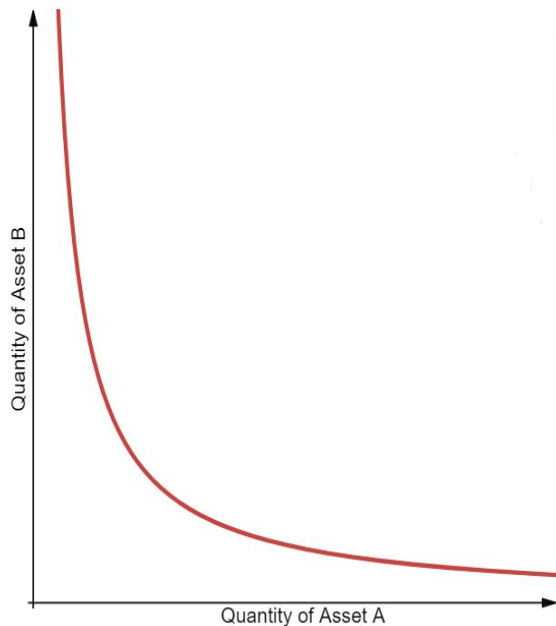**I sold my 1 WBTC for 19 ETH (38,000 USD)**

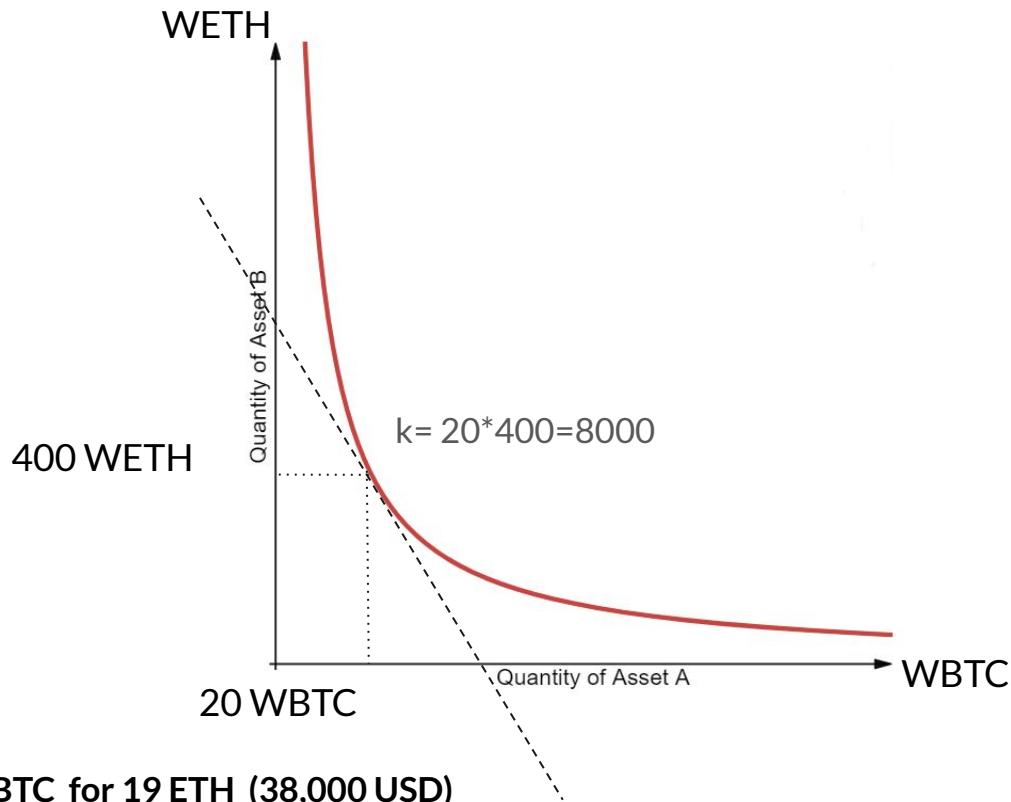# Constant Product Market Making by example
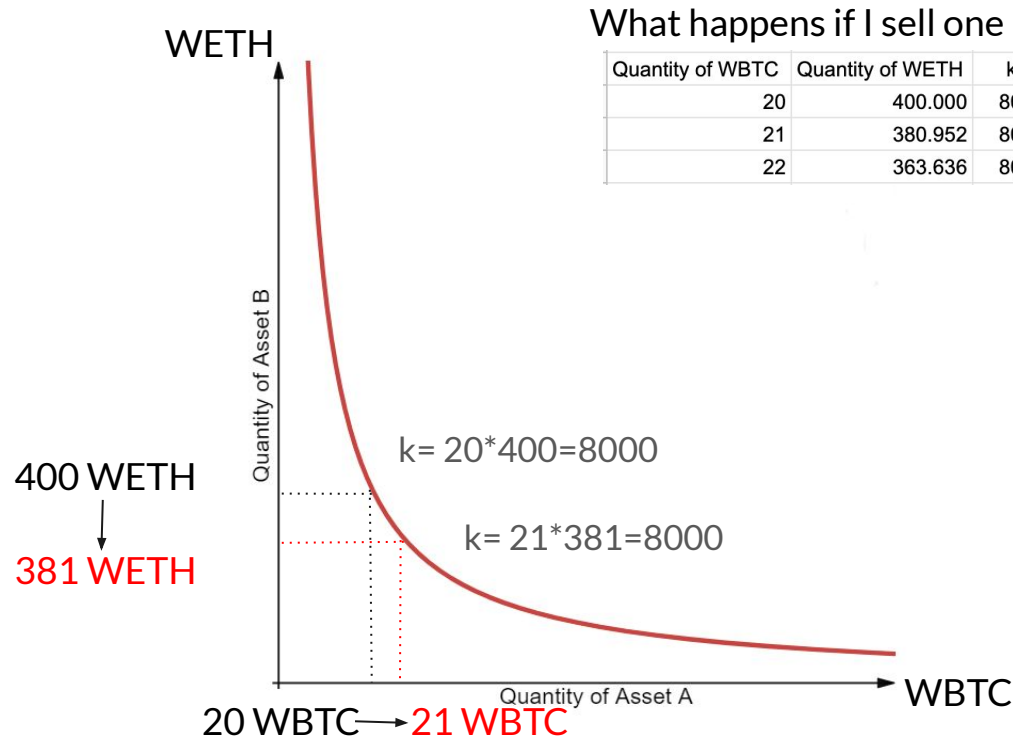
What happens if I sell one more WBTC?

Quantity of WBTC	Quantity of WETH	k	How much WETH do I get when I **sell 1 more WBTC**?	(Assuming 1 WETH = 2000 USD)
20	400.000	8000	19.0476190	38095.2380952
21	380.952	8000	17.3160173	34632.0346320
22	363.636	8000		

WETH

400 WETH

381 WETH

k= 20*400=8000

k= 21*381=8000

Quantity of Asset B

Quantity of Asset A

WBTC

20 WBTC → 21 WBTC

Constant Product Market Making by example

What happens if I just kept on selling WBTC?

Quantity of WBTC	Quantity of WETH	k	How much WETH do I get when I **sell 1 more WBTC**?	(Assuming 1 WETH = 2000 USD)
20	400.000	8000	19.0476190	38095.2380952
21	380.952	8000	17.3160173	34632.0346320
22	363.636	8000	15.8102767	31620.5533597
23	347.826	8000	14.4927536	28985.5072464
24	333.333	8000	13.3333333	26666.6666667
25	320.000	8000	12.3076923	24615.3846154
26	307.692	8000	11.3960114	22792.0227920
27	296.296	8000	10.5820106	21164.0211640
28	285.714	8000	9.8522167	19704.4334975
29	275.862	8000	9.1954023	18390.8045977
30	266.667	8000	8.6021505	17204.3010753
31	258.065	8000	8.0645161	16129.0322581
32	250.000	8000	7.5757576	15151.5151515
33	242.424	8000	7.1301248	14260.2495544
34	235.294	8000	6.7226891	13445.3781513
35	228.571	8000	6.3492063	12698.4126984
36	222.222	8000	6.0060060	12012.0120120
37	216.216	8000	5.6899004	11379.8008535
38	210.526	8000	5.3981107	10796.2213225
39	205.128	8000	5.1282051	10256.4102564
40	200.000	8000		

Quantity of Asset B

Quantity of Asset A

Jargon: slippage

Constant Product Market Making by example

WETH

Quantity of Asset B

400 WETH

k= 20*400=8000

Quantity of Asset A

WBTC

20 WBTC

I sold my 1 WBTC for 19 ETH (38,000 USD)

Constant Product Market Making by example
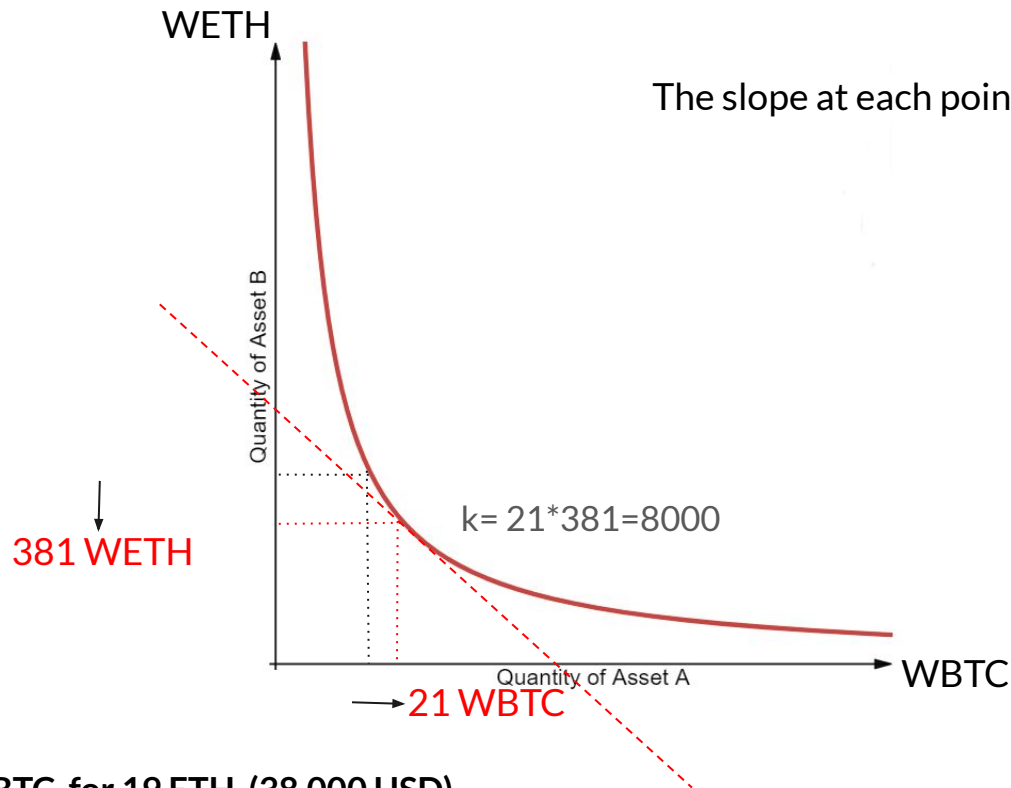
WETH

The slope at each point is exactly the **marginal** exchange rate

Quantity of Asset B

381 WETH

k= 21*381=8000

Quantity of Asset A

21 WBTC

WBTC

I sold my 1 WBTC for 19 ETH (38,000 USD)

Constant Product Market Making by example

WETH

Assume the fair market price elsewhere is
1 WBTC = 40,000 USD, 1 WETH = 2,000 USD

Is WBTC now cheap or expensive?
What will happen now?

Quantity of Asset B

k= 21*381=8000

381 WETH

Quantity of Asset A

WBTC

21 WBTC

I sold my 1 WBTC for 19 ETH (38,000 USD)

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381 WETH

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Quantity of Asset B

Quantity of Asset A

WBTC

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Constant Product Market Making by example

WETH

Quantity of Asset B

400 WETH

381 WETH

k= 20*400=8000

k= 21*381=8000

Quantity of Asset A

20 WBTC ← 21 WBTC

WBTC

Assume the fair market price elsewhere is
1 WBTC = 40,000 USD, 1 WETH = 2,000 USD

Is WBTC now cheap or expensive?
WBTC is cheap, WETH is expensive

What will happen now?
Arbitrageurs will sell WETH for WBTC (i.e. buy WBTC)
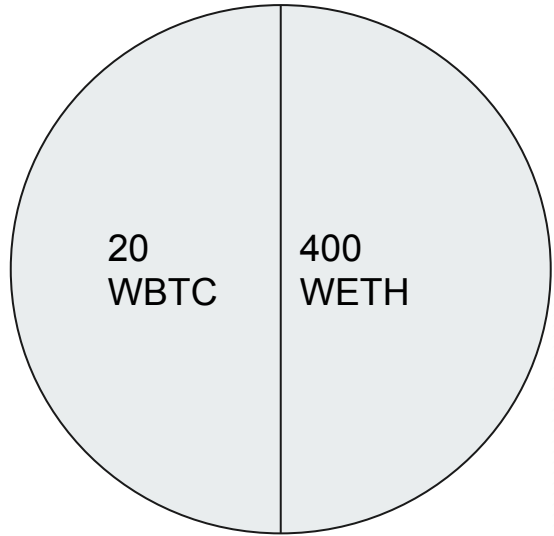e.g. Someone will sell 19 WETH to get 1 WBTC

Liquidity Provider: What is adding/removing liquidity?
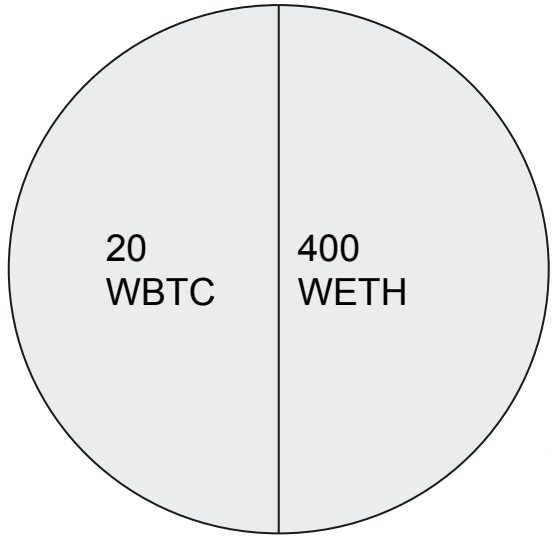
I am the one who provided all the capital for people to trade against, how do I prove that I am a market maker?

20 WBTC

400 WETH

Liquidity Provider: What is adding/removing liquidity?

Liquidity Provider(LP) Token

20
WBTC

400
WETH

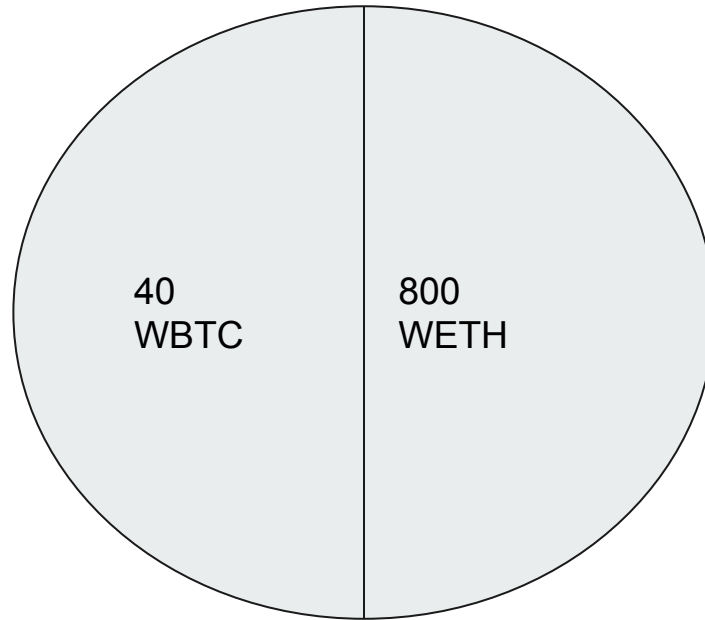I have 1 LP Token, the total supply of LP Token is 1, therefore I own 100% of the WBTC-WETH pair

Liquidity Provider: What is adding/removing liquidity?

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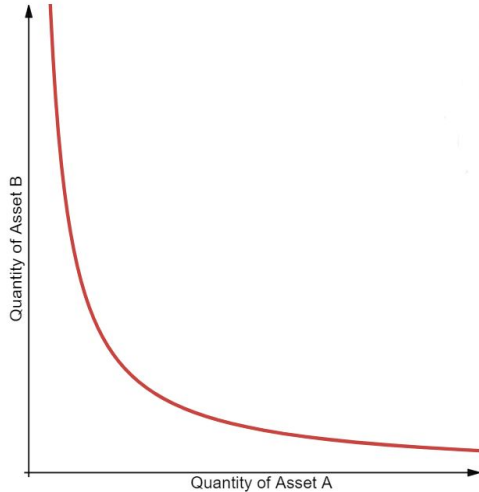
Smart contract mints a new LP token, the total supply of the LP token is now 2, which means each is worth 50% of this trading pair

40
WBTC

800
WETH
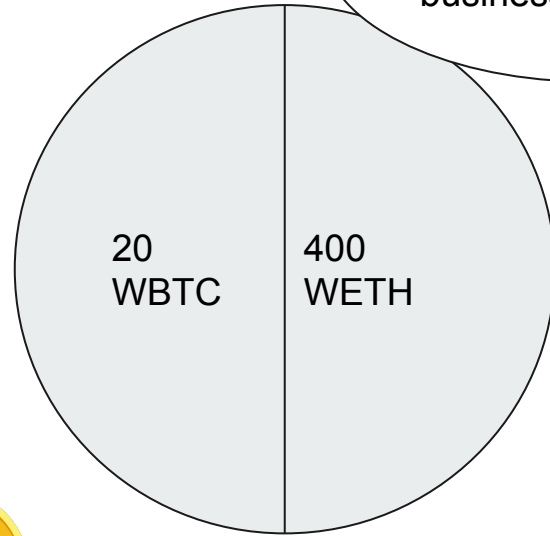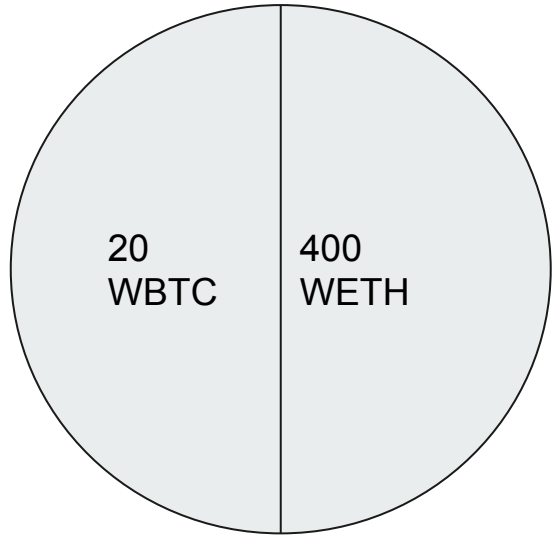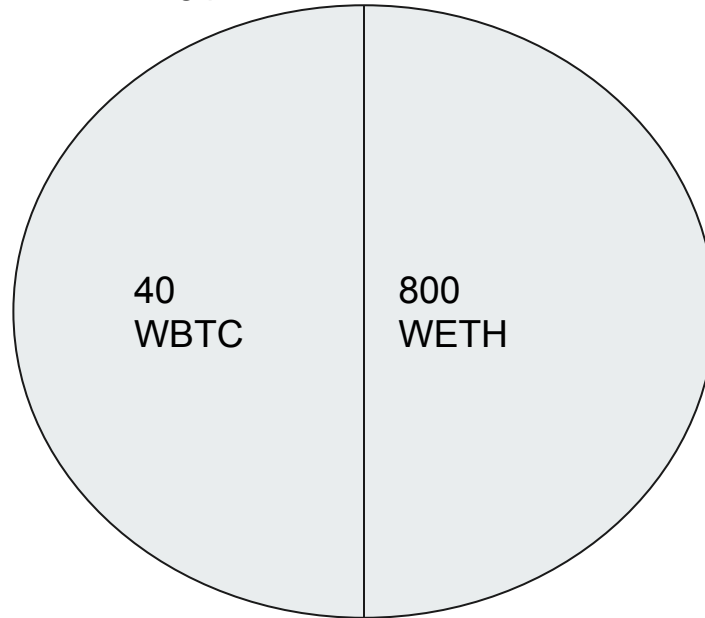
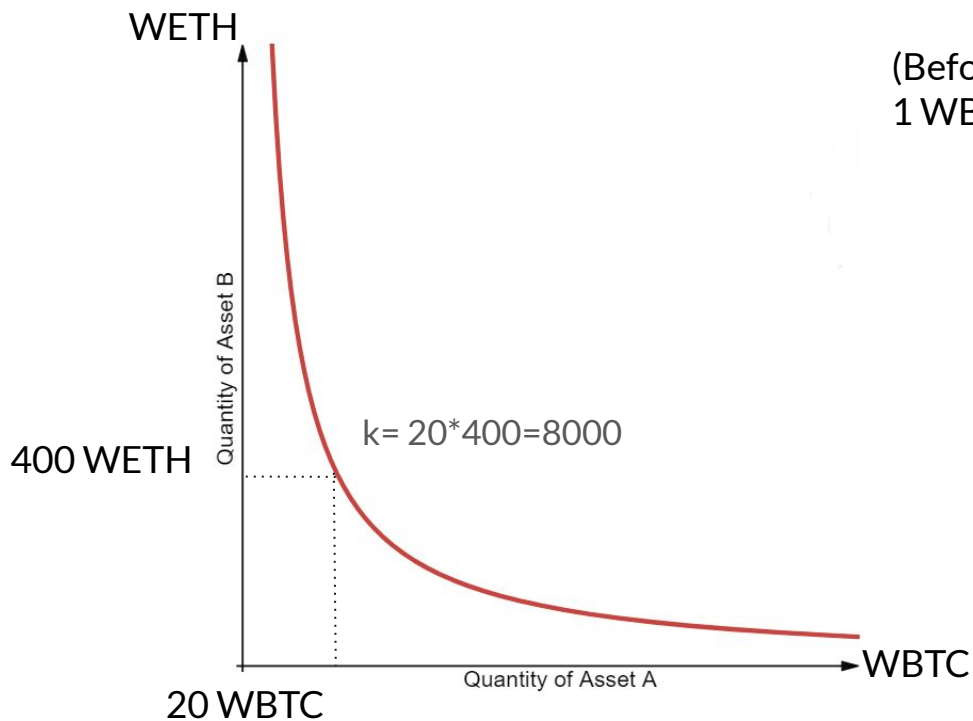Let's partner up and run this business together!

Jargon: Liquidity pools

Liquidity pools are smart contracts that hold balances of two unique tokens and enforces rules around depositing and withdrawing them.
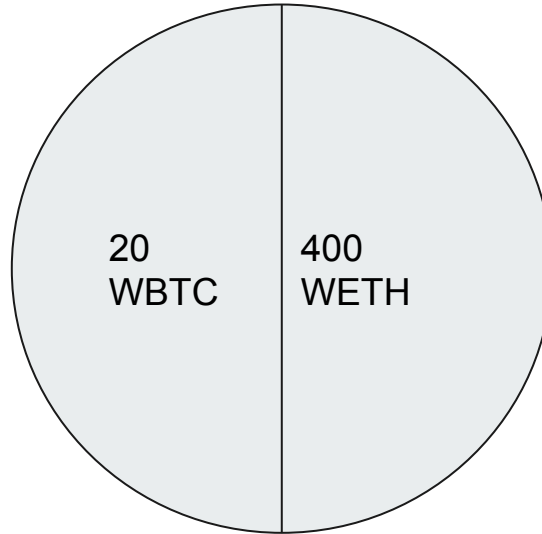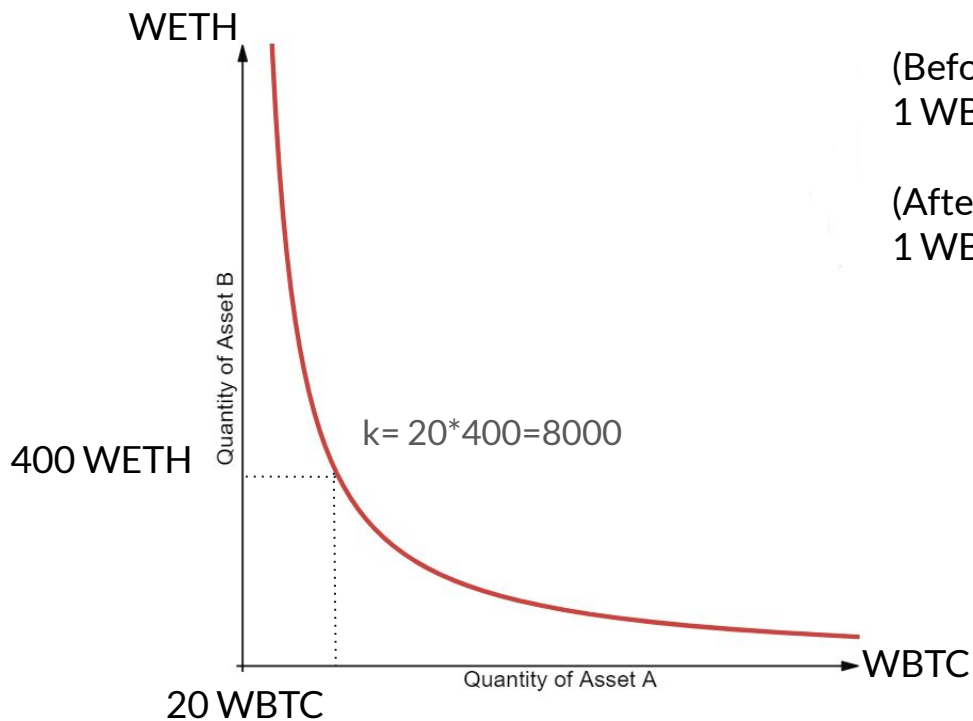
What if the market price changed?

WETH

Quantity of Asset B

k= 20*400=8000

400 WETH

20 WBTC

Quantity of Asset A

WBTC

(Before) the fair market price elsewhere is
1 WBTC = 40,000 USD, 1 WETH = 2,000 USD

What if the market price changed?

WETH

Quantity of Asset B

400 WETH

k= 20*400=8000

Quantity of Asset A

WBTC

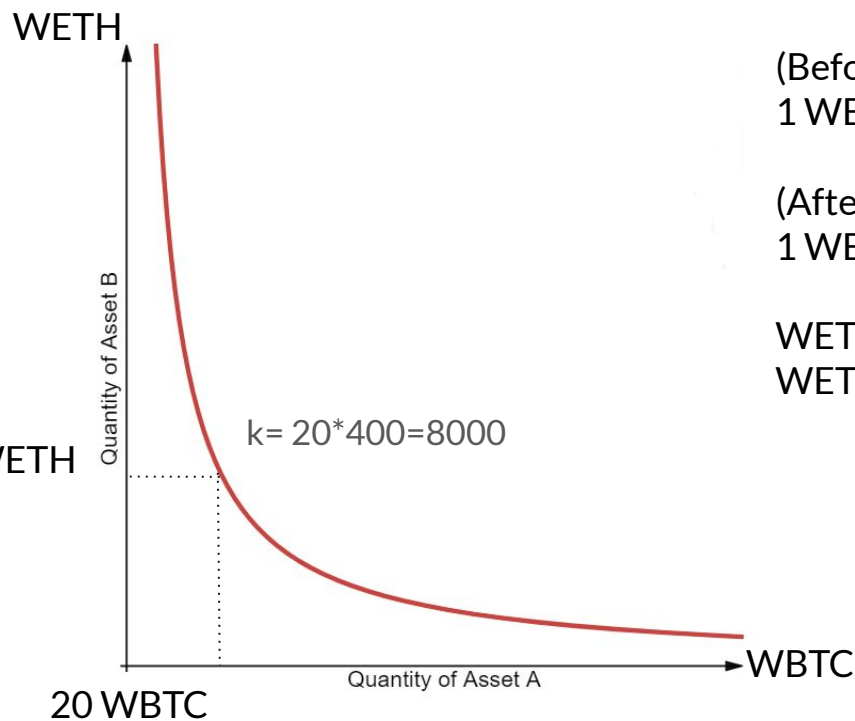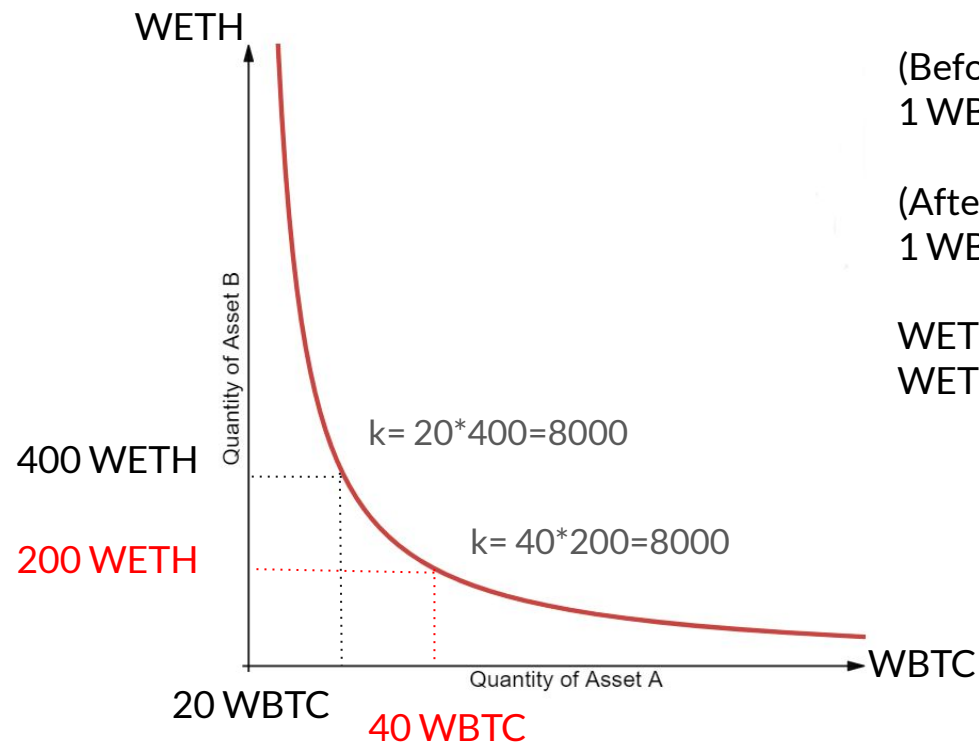20 WBTC

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(After) 🚀 🚀 🚀 💎 💎 💎 📈 📈 📈
1 WBTC = 40,000 USD, 1 WETH = 8,000 USD

What if the market price changed?

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WETH price is cheap, arbitrageurs will sell WBTC to get
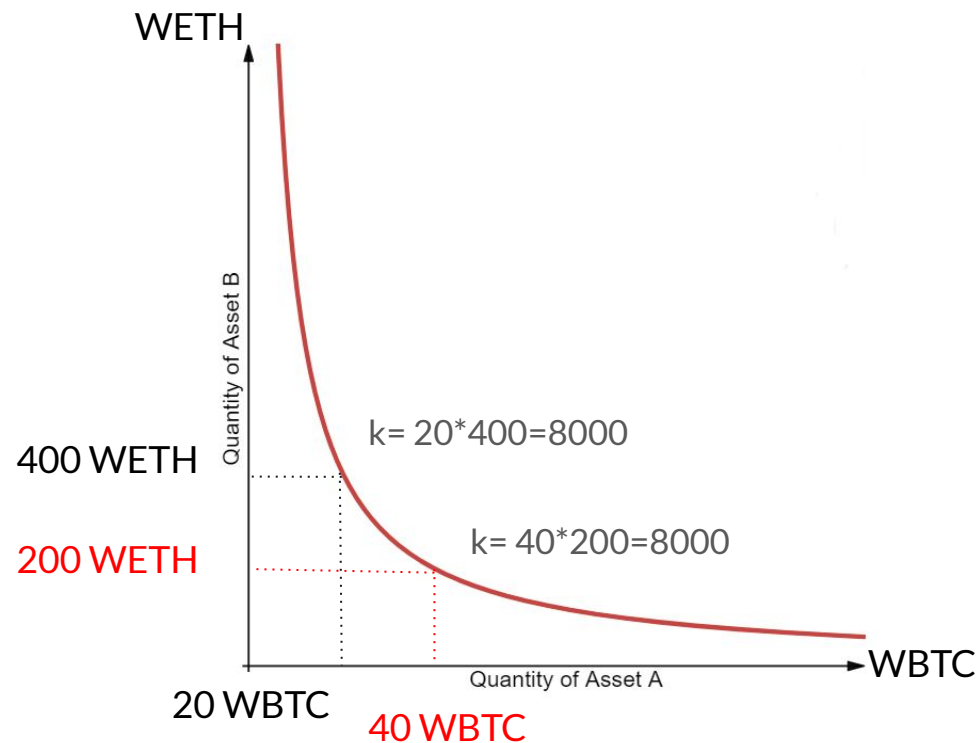WETH, so that can sell elsewhere

What if the market price changed?

WETH

Quantity of Asset B

400 WETH

200 WETH

k= 20*400=8000

k= 40*200=8000

20 WBTC

40 WBTC

Quantity of Asset A

WBTC

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(After) 🚀 🚀 🚀 💎💎💎 📈📈📈
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WETH, so that can sell elsewhere

What if the market price changed?

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Quantity of Asset B

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400 WETH

200 WETH

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Quantity of Asset A

WBTC

20 WBTC
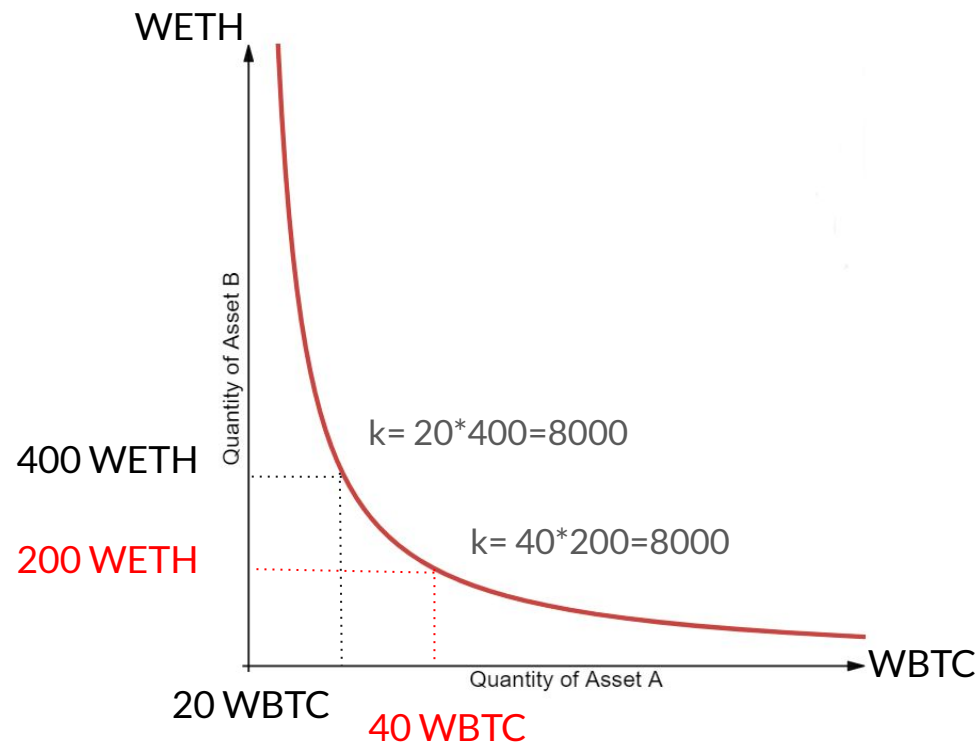
40 WBTC

(Before)
1 WBTC = 40,000 USD, 1 WETH = 2,000 USD

-We had 20 WBTC, 400 WETH =800k+800k= 1.6m
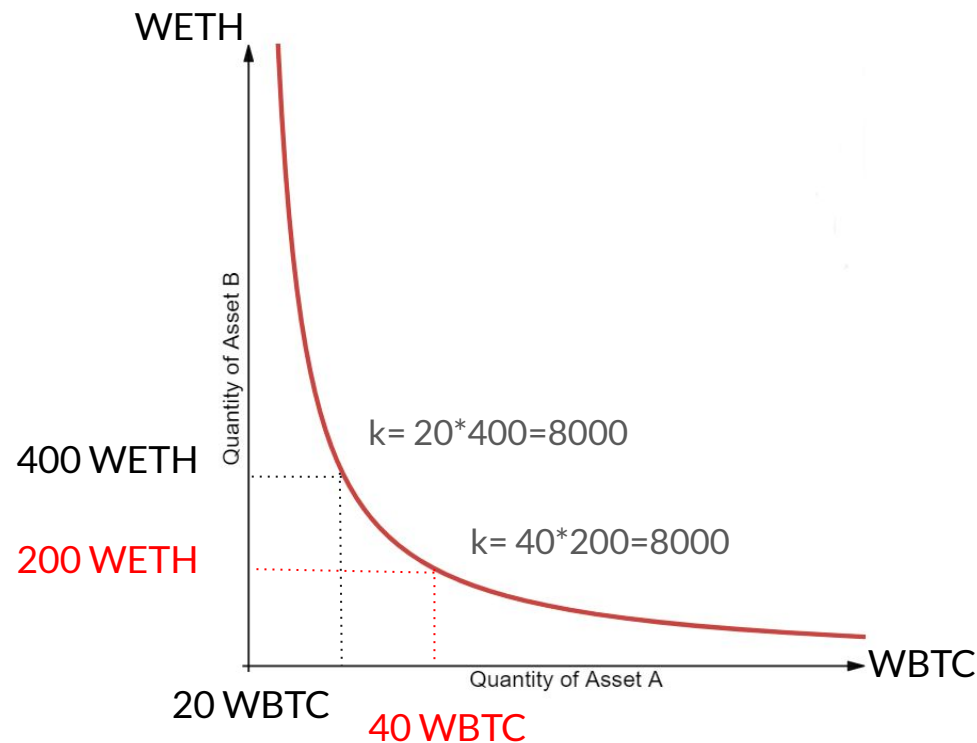
(After) 🚀🚀🚀 💎💎💎 📈📈📈
1 WBTC = 40,000 USD, 1 WETH = 8,000 USD

-Pool now has 40 WBTC, 200 WETH = 3.2m

What if the market price changed?

WETH

Quantity of Asset B

k= 20*400=8000

400 WETH

200 WETH

k= 40*200=8000

Quantity of Asset A

20 WBTC

40 WBTC

WBTC

(Before)
1 WBTC = 40,000 USD, 1 WETH = 2,000 USD

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🤑

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WETH

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If do nothing and HODL:
-Still have 20 WBTC, 400 WETH = 800k+3.2m=4m!

What if the market price changed?

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What happened?

We originally had 1.6m worth of tokens.
WETH price went up by 300%

Pool net value went from 1.6m to 3.2m, made 100%

If we did nothing and HODL: 1.6m to 4m, could have made 150%

We "lost" 0.8m, which is 0.8/4 =20%

What if the market price changed?

(Before)
1 WBTC = 40,000 USD, 1 WETH = 2,000 USD

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(After) 🚀🚀🚀 💎💎💎 📈📈📈
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How does a Liquidity Provider make money?

1. Capital gains
   - However, "impermanent loss"
   - Mean-reverting assets: USDC/DAI, or WBTC/renBTC generally have the lowest IL

2. Transaction fees
   - 0.3% for Uniswap
   - have the option to turn on 0.05% protocol fee

3. (optional) Additional incentives like token airdrops
   - i.e. yield farming, liquidity mining

Notice how if you assume all trades are arbitrage transactions(informed flow), the pool loses money on every trade.

Recap: What is Market Making?

Manual

1. Traditional Commerce

   E.g. Airport currency exchange

Algorithmic

2. Traditional Finance

   E.g. Citadel Securities
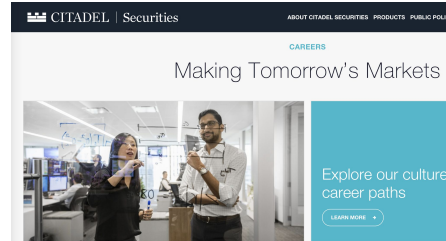
Decentralized

3. **Decentralized Finance**

   E.g. Uniswap

How do market makers make money?

1. Spread
2. Designated market making arrangements
   - typically paid by asset issuers
3. Fee rebates
   - typically paid by an exchange

How do market makers make money?

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   - typically paid by asset issuers
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HKEX 香港交易所 **MARKET MAKING ARRANGEMENTS OF EXCHANGE TRADED PRODUCTS**

Market maker incentives
- Market makers enjoy the following benefits:
  - Market making trades are exempted from trading fee (0.005%) and transaction levy (0.0027%);
  - Market makers can apply for Market Making Orion Central Gateway (MMOCG) Session at a discounted rate for market making of ETPs.

WSB: How does Robinhood make money?

Pay for order flow

Informed flow

- Arbitrage transactions

Uninformed flow

- Random

This is why Citadel will pay for order flow from Robinhood, which is mainly retail uninformed flow

Notice how Uniswap can't tell the informed from uninformed, every trade is treated as equals

x-y-k AMMs like Uniswap

1. Faster exchange
2. Simply and elegant
3. Friendly to low liquidity assets
4. Easy to bootstrapping liquidity
5. AMMs as on-chain price oracles
   - Caveat: security concerns
   - See Uniswap v2 TWAP
6. Significantly lower setup cost
7. Clearly better for stablecoins
   - Mean -reverting: USDC/DAI
   - Uncorrelated: ETH/DAI

1. Slippage
2. Financial risk
   - i.e. Impermanent Loss implies that Uniswap's payoff curve is concave

3. Smart Contracts are not smart (yet)

   Like a vending machine that only knows $x*y=k$

   =>good for those who knows how to manage risk

   =>bad for unsophisticated and passive MMs

Advanced topics:

1.   Some Intriguing Results

"If the volatility of an asset is high enough relative to its average rate of return, LPs on Uniswap will do better than HODLers over time, *even when the only incoming trades are arbs*"

Dave White & Martin Tassy:
https://math.dartmouth.edu/~mtassy/articles/AMM_returns.pdf

2.   AMMs aren't new

AMMs have actually been used in prediction markets for a while. Arguably the most famous(and the most widely implemented) one is Hanson's LMSR market maker for prediction markets

Robin Hanson: https://mason.gmu.edu/~rhanson/mktscore.pdf

This talk:

Decentralize

This talk:

Decentralize

Smart Contract

This talk:

Decentralize

Appendix: Why aren't order books popular on-chain (yet) ?

1. The size of the state needed by an order book to represent the set of outstanding orders (e.g., passive liquidity) is large and extremely costly in smart contracts

    -users must pay for space and compute power utilized

2. The matching logic for order books is often complicated as it must often support different order types:
    a. Good-till-cancel
    b. Stop-limit

3. Keeping order book state in the hands of multiple parties: latency arbitrage

References:

- An analysis of Uniswap markets: https://arxiv.org/abs/1911.03380
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- Logarithmic Market Scoring Rules for Modular Combinatorial Information Aggregation: https://mason.gmu.edu/~rhanson/mktscore.pdf
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